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DETERMINATION OF REASONABLE FIT INTERFERENCES FOR SOLID WOOD JOINTS USING FINITE ELEMENT METHOD (FEM)

Tibor PAPP[1], Zsolt KOVACS[1]
[1]Institute of Product Development and Manufacturing
Faculty of Wood Sciences
University of West Hungary

Abstract:
Performance of joints of solid wood parts such as different types of mortise and tenon joints depends much on the tightness of fit between tenon and mortise. Generally interference between tenon and mortise thickness is needed for a good load bearing capacity of the joint, the extent of which depends on wood species. A limiting factor for fit tightness is the development of tensile stresses across grain direction around the corners. These stresses can be modelled by finite element technique using orthotropical material models. The objective of this paper is to define reasonably applicable interferences in the case of joints of different wood species on the basis of across-the-grain tensile stresses introduced in the solid wood parts when assembling joints. Finite element models elaborated for the joints studied are verified by physical testing. Tests extend to assessing the performance of the joints.

Keywords: *solid wood, tenon, mortise, interferences, finite elements method (FEM)*

INTRODUCTION

Problem stating

The topic of this paper is the study of applicable fits in woodworking in the case of furniture joints. More specifically, we are interested in the reasonable extremes of fit interferences that influence tolerances to be specified for parts making the joint, that is tenon and mortise respectively in open or "slip or lock-type" corner joints. Besides trying to give an overview and appraisal of the fit and tolerance system in relation with woodworking industry we model the behaviour of joints of varying fit interference with a view to their mechanical strength and the risk of occurring failures in the wood parts due to assembly. An additional purpose of modelling by using physical and virtual models is to explore the applicability of finite element methodology to wood as an orthotropic material.

Theoretical background

Tolerances and fits in the woodworking industry had traditionally been determined in a manner similar to what was used in making metal parts. However particular features of wood as raw material have not been properly taken into consideration. Besides, processing technologies developed considerably; as a consequence, manufacturers could not use successfully those values of fits tolerances set in the relevant standard, leading to the alleviation of the same toward the end of 1990s. To date, there exist no valid standard for the Hungarian wood-processing industry. At the same time, in branches like plastics and metal processing new concepts of establishing tolerances are successfully applied. For the woodworking industry the five aspects of the former system as below should be kept in mind when working out a new, viable system of fits and tolerances:

1. Functional requirements and economical consequences had not been articulated.
2. Higher classes of precision could not be realised in practice.
3. Verification of the numerical values is not straightforward and cannot be interpreted in the case of up-to-date machines.
4. Precision and applicable tolerance is influenced by a number of factors other than those related to the machine.
5. Measurement of part dimensions holds uncertainties due to the surface structure of wood.

In the production process, all the problems that can be attributed to the raw material will exhibit themselves as a result of a number of features. These sets of features can be best dealt with if we establish raw material categories based on wood species and try to find the optimal machining parameters for these categories.

In this paper we study the effects that the different species of wood, through their density, texture, porosity and strength properties may have on the applicable amount of fit and tolerances for making structural joints.

Up-to-date concepts and methods in tolerance design

Up-to-date tolerance design is based on the joint application of several concepts. Engineering tolerances have a role in three areas of product manufacturing: in product design, in the manufacturing process and in quality control. The greatest challenge is to determine and incorporate the customers-driven tolerances into the product, than to interpret the same as the tolerances of a controlled and optimised manufacturing process. The approach that can be proposed for setting tolerances may be as follows:

1. Clarifying the functional requirements (functionally approvable limits of precision).
2. Determining the precision that can be realised by the machines and processes (process capability).
3. Assessing the economical consequences.

From the point of view satisfaction of functional requirements on reasonable costs is important. When point 2) or 3) is an obstacle, than the construction of the product has to be modified, that is made mor robust int he sense that a higher tolerance would not compromise the function.

As regards tolerance design one can find three approaches in the literature:

Worst Case Analysis (WCA) **–** Tolerances on stacks (assembled units) are determined from the nominal dimensions and tolerances of parts in a way that allowable extremes of part dimensions are summed up. *Root Sum of Square (RSS*) **–** The distribution of the parts' dimension within their tolerance limits is taken into consideration. This distribution is taken as normal and can be described by means of the standard deviation derived from the process capability index. Using this method the probability of any values of clearance (gap) or interference (overlapping of tenon and mortise dimensions) in a fit can be calculated. *Taguchi Tolerance Design (TTD)* **-** As input of tolerance design we use the output of parameter design, that is the optimal setting of design parameters determining the fulfilment of functional requirements. The method builds on the sensitivity of functional characteristics on the tolerance of design parameters. That is, it looks for those design parameters the tolerance of which has the most influential effect on the variability of the performance of the product. Analysis of variances is a useful tool for this task.

Distribution of the gaps and interferences of fits used in the woodworking industry

Wood products assembled from parts apply both loose and tight fits. Theoretical values of gaps and interferences as well as tolerances of fits can be calculated by means of the expressions as follows

$$J_{max} = A_s - a_i \qquad J_{min} = A_i - a_s \qquad [1][2]$$

$$S_{max} = a_s - A_i \qquad S_{min} = a_i - A_s \qquad [3][4]$$

$$T_c = A_s - A_i \qquad T_s = a_s - a_i \qquad [5][6]$$

$$T_{aj} = T_s + T_c = S_{max} + J_{max} \qquad [7]$$

where:

A_s, A_i – lower and upper specificattion limits for the mortise,

a_s, a_i - lower and upper specificattion limits for the tenon,

J_{max}, J_{min} – maximum and minimum of gaps,

S_{max}, S_{min} – maximum and minimum of interferences,

T_c, T_s, T_{aj} – tolerances for tenon, mortise and fit respectively.

Distribution of the gaps and interferences. In the case of manufacturing processes that are statistically in control the dimensions of the individual tenons and mortises follow normal distribution with standard deviations of σ_c and σ_s respectively. Therefore the ranges of variation for these dimensions can be characterised by bandwidths of 6σ. The same is true for the distribution of gaps and interferences occurring in joints characterised by σ_{joc} and $\sigma_{str.}$

Since the real dimensions of tenons and mortises are results of a number of random effects and are independent from each other, it can be written for the standard deviation of the gaps and interferences that

$$\sigma_{aj.pr} = \sqrt{\sigma_s^2 + \sigma_c^2} \qquad [8]$$

Given that $6\sigma_{aj} = w_{aj}$, $6\sigma_s = T_s$, and $6\sigma_c = T_c$, the range of variation of the fit gap/interference is:

$$w_{aj} = \sqrt{T_s^2 + T_c^2} \qquad [9]$$

This interval corresponds to the practical tolerance, therefore it can be written:

$$T_{aj.pr} = \sqrt{T_s^2 + T_c^2} \qquad [10]$$

It can be seen that the practical interval is less than the theoretical tolerance, that is:

$$T_{aj.pr} = \sqrt{T_s^2 + T_c^2} < T_{aj} = T_c + T_s \qquad [11]$$

As a consequence, the practical extremes of gaps and interferences differ from the theoretical values as given by the expressions below:

$$J_{max..pr} = J_{max} - (T_{aj} - T_{aj.pr})/\ 2 \qquad J_{max..pr} = J_{max} - (T_{aj} - T_{aj.pr})/2 \qquad [11][15]$$

$$S_{max..pr} = S_{max} + (T_{aj} - T_{aj.pr})/\ 2 \qquad S_{max..pr} = S_{max} - (T_{aj} - T_{aj.pr})/2 \qquad [12][16]$$

$$J_{min..pr} = J_{min} + (T_{aj} - T_{aj.pr})/\ 2 \qquad J_{min..pr} = J_{min} + (T_{aj} - T_{aj.pr})/\ 2 \qquad [13][17]$$

$$S_{min..pr} = S_{min} - (T_{aj} - T_{aj.pr})/\ 2 \qquad S_{min..pr} = S_{min} + (T_{aj} - T_{aj.pr})/\ 2 \qquad [14][18]$$

From the above expressions it can be seen that both the practical tolerance intervals of fits and the practical ranges of gaps and interferences are inferior to the theoretical values. The question can be put up inversely: the practical tolerances for the tenon and mortise should be determined in a way that they result the desired looseness or tightness of fit. In cases when machining precision, hence tolerances for tenon and mortise are identical, the theoretical tolerance for the fit results in $T_{aj}=T_c+T_s=2T_c$. From the same formula it follows that tolerance for the tenon and mortise is $T_c=T_s=T_{aj}/2$.

By virtue of the normal distribution of dimensions the practical tolerance of the fit will be:

$$T_{aj.pr} = \sqrt{T_c^2 + T_s^2} = \sqrt{2 \cdot T_c^2} = \sqrt{2 \cdot T_s^2} = T_s\sqrt{2} = T_c\sqrt{2} \qquad [19]$$

$$T_{c.pr} = T_{s.pr} = T_{aj.pr}\ /\sqrt{2} \qquad [20]$$

When the practical tolerance of fit equals the theoretical one, the tolerances for the tenon and mortise will increase.

Practice shows that the narrower the tolerance interval of a part and the larger its dimension, the more complicated and more expensive becomes its manufacturing technology. The required quality of products, the safety of the manufacturing process as well as the keeping low the production costs can be assured by grouping the products into standard precision classes. In practice these goals can be achieved by working with tolerances increased to some degree. It can be realised more easily, what is more, interchange ability of parts can be can be assured at lower costs. In the case of wood it can also be established, that surface quality of faces to be fitted has an effect on the ranges of interference to be used.

Load-bearing capacity of the joint is influenced by fit clearance/interference. In general it can be said that a joint with tighter fit is mechanically superior to those of loose fit or zero clearance. Therefore, precision of manufacture (nominal values and tolerances) for the parts should be specified to assure the required amount of interference. At the same time, we have clarify the maximum value of interference the wood part is able to assume without damage.

This is the most important question in our study that we try to find answer to by simulating the stressed state of joint parts by means of both physical and virtual models,

EXPERIMENTAL

Virtual modelling

Test pieces of tenon and mortise joint made of spruce (Picea *abies*) were modelled for conducting Finite Element Analysis (FEA) of the stresses developing due to fit interference by using SolidWorks Simulation. The material properties in air-dry condition (12 % M.C.) are as below:

- compression parallel to the grain: 35-50-79 MPa
- bending strength: 49-78-136 MPa
- tensile strength: 21-90-245 MPa
- shear strength: 4-6,7-12,0 MPa
- modulus of elasticity in bending: 7300-11000-21400 MPa

Orthotropic elastic properties in static loading, used in the FEA model:

- E_L: 10000 MPa
- E_R: 860 MPa
- E_T: 518 MPa
- G_{LR}: 735 MPa
- G_{LT}: 691 MPa
- G_{RT}: 70 MPa
- μ_{LT}: 0,42
- μ_{LR}: 0,37
- μ_{RT}: 0,47
- μ_{RL}: 0,041
- μ_{TR}: 0,35
- μ_{TL}: 0,033

The FEA model of the joints contained the symmetrical half of thickness of test pieces as shown in Figure 1 below. Different degrees of fit tightness were introduced in the model by defining shrink fit contact of different interference values between the tenon and mortise face.

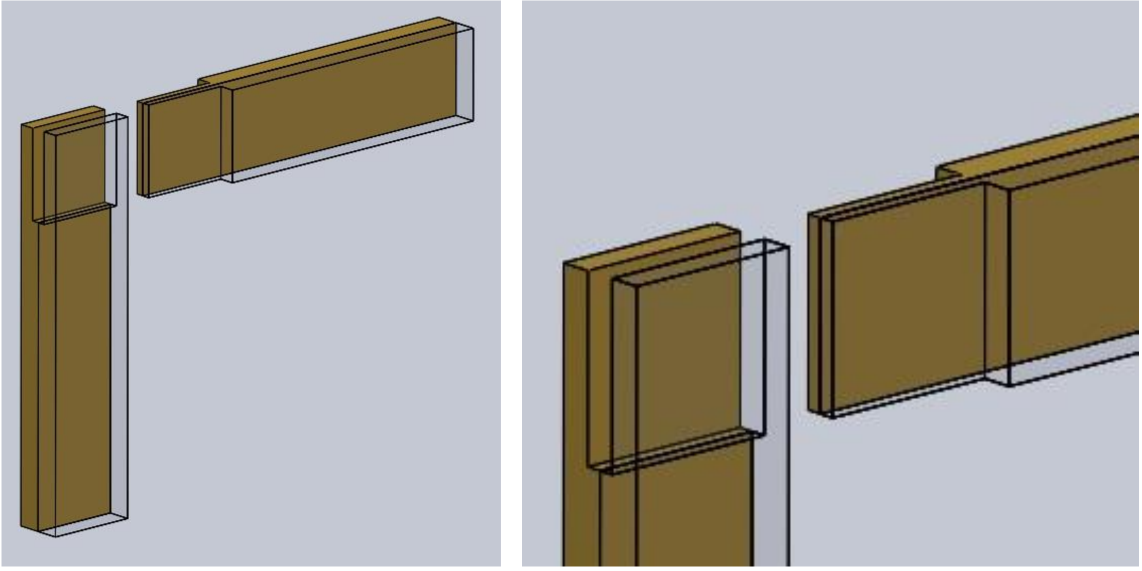


Figure 1. The FEA model of joints

Figure 2. illustrates the normal stresses in the through-thickness (i.e. perpendicular to the grain) direction of the test piece. It is clearly demonstrated that high tensile stresses appear along the re-entrent corner in the mortise piece concentrating in a very narrow area. However, when these stresses attain a critical value that can be related to the tensile strength of the wood in the perpendicular-to-the-grain direction, are likely to cause to fail the piece in the corner. Peak value can be observed in the inside section of the corner line, the effect of which extenuates to some degree towards the edge of test piece. Therefore it is more reasonable to evaluate the magnitude of these tensile stresses on the side face of the mortise piece along to the corner. Besides, since stress values decay in a very rapid manner with the distance from the corner line, their absolute value can not be directly taken as a measure of failure indicator. For a full evaluation of the stresses resulted from the finite element analysis two additional facts has to be considered in the case of parts made of wood. The first is, that the machined surface structure of wood, when two pieces are pressed against each other, gives room for compression of rough surfaces first with relatively low resistance. Secondly, high local stresses tend to relax with time due to the visco-elastic nature of wood. Unfortunately, the available FEA software package do not support this typa of material behaviour in the case of an orthotropic material model. At the same time, their relative values at varying interferences may give an indication on how the risk of failure increases with tightening the fit. Evaluation of the model results may be complemented by comparison with experimental observations.

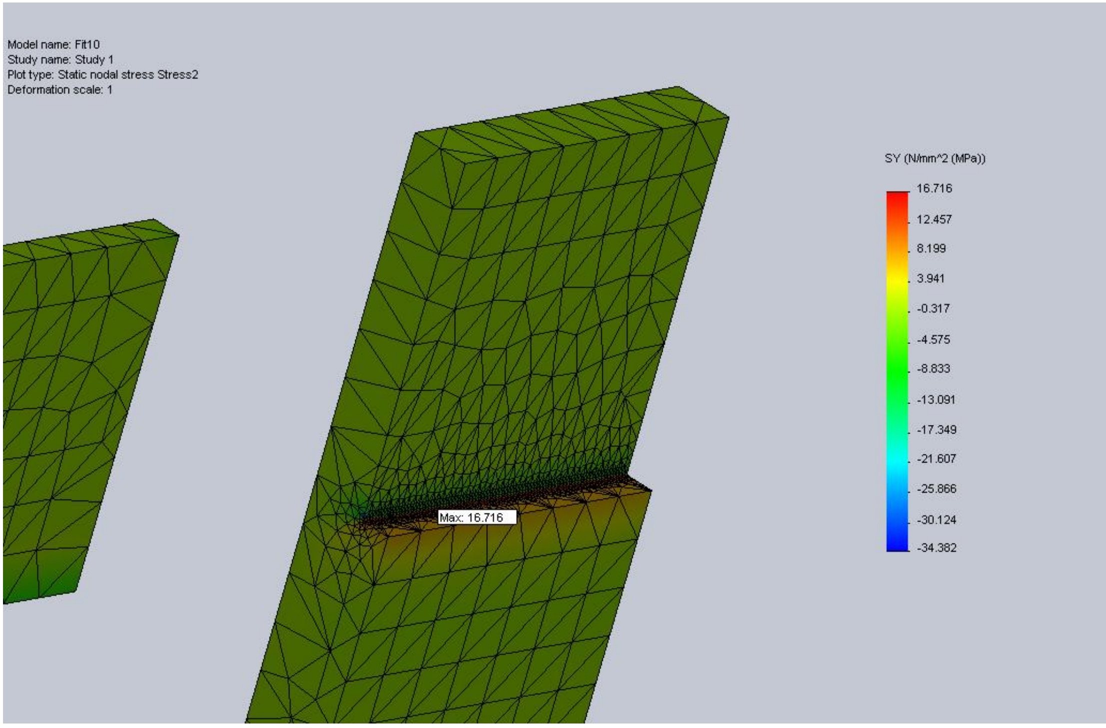


Figure 2. Perpendicular-to-the-grain tensile stresses in the mortise corner line detected by finite element analysis in an open mortise and tenon joint with 0.1 mm interference

The size of interference in the shrink-fit model corresponds to a two-times as much difference between the corresponding dimensions of the tenon and mortise respectively before assembly. For example, an interference value of 0.1 mm in the case of a tenon of 8.0 mm nominal thickness means that the actual tenon thickness is 8.2 mm while the mortise is 8 mm thick.

Figure 3 illustrates the dependence of the peak value, as well as the value at the corner of across-the grain tensile stresses determined from FEM analysis on the size of interference.

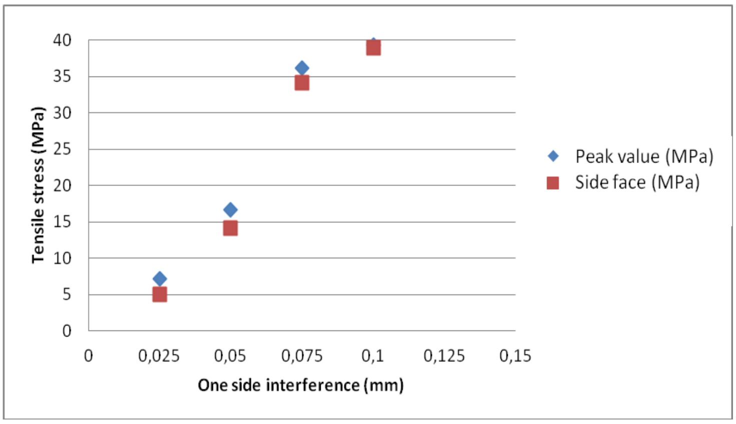


Figure 3. Peak tensile stress values and stress values along at the side of mortice piece by changing the amount of interference

EXPERIMENTING WITH REAL SPECIMENS, MODEL VALIDATION

Model validation

The density of spruce species (*Picea abies*) is highly dependent of production site, the measured values range from 300-430 to 640 kg/m3 for completely dry wood, from 330-470 to 680 kg/m3 for air-dried wood and it is 740 kg/m3 in green state. The radial shrinkage of spruce is 3,6%, the tangential shrinkage is 7,8% and it is about 0,3% in longitudinal direction. The volumetric shrinkage is 12,0%.

Specimen preparation

The specimen's dimensions were determined in function of the standard corner joint test procedures and testing machine parameters. Specimen's size was 24 mm in thickness, 40 mm in width and the length was 300 mm. The thickness of the mortise and tenon was exactly one third of the wood. The mortise thickness was fixed at a value of 8 mm, while the tenon thickness increased from 8 to 10 with steps of 0,5 mm. Increasing the tenon thickness resulted in an artificially overstressed connection. Moisture content of specimens was conditioned at 12%.

For specimens preparation we used the conventional machining technology. The manufacturing was preceded by careful selection, in which species-specific properties, uniform structure, same annual ring position and defect elimination were considered. After the cross section profiling and cutting to the final length, the mortise and tenon geometry were formed using a tenon forming machine. After machining the average mortise thickness was 8.07 mm with a standard deviation of 0.040 mm, maximum and minimum of 8.15 7.99 mm the measured values followed a normal distribution.

Joint assembling

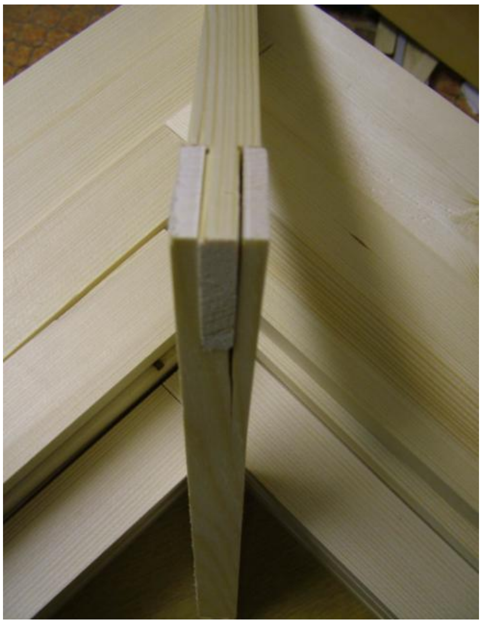
The prepared specimens were conditioned in the wood shop for 2 weeks, while the moisture content decreased to 10,5%. This resulted in the shrinkage of the tenons from 8,07 mm to 8,04 mm, however, the 0,0037% change were considered not significant and not influenced considerable the results. The mortise moisture content and changed dimensions are included in Table 1.

Table 1: The change of mortise thickness

Nr.	at machining M.C.= cca. 12%	after conditioning M.C.= cca. 10,5%			Change (mm)
1.	**8.13**	8.02	8.14	**8.08**	-0.05
2.	**8.11**	7.96	8.02	**7.99**	-0.12
3.	**8.03**	7.96	8.06	**8.01**	-0.02
4.	**8.12**	7.98	8.16	**8.07**	-0.05
5.	**8.15**	8.08	8.16	**8.12**	-0.03
6.	**8.03**	7.98	8.02	**8.00**	-0.03

In the case of tenons the shrinkage caused a thickness decrease of 0.019-0.0579 which is not significant also and does not influence the results' evaluation. After the mortise and tenon preparation a pairing procedures took place to cover best the pre-stressed range. The overlapping values fell between an interval of 0.16 to 2.15 mm. After assembling the selected mortise and tenon pairs the following visual results can be concluded:

- in the case of 8 and 8.5 mm thick tenons no change or cracks were observed
- when the tenon thickness was 8.5 mm the +0.5 mm oversize dispersed evenly because of wood elasticity and did not increased the joint thickness
- at 9; 9.5; 10 mm nominal thickness values immediately after assembling cracks appeared starting from tenons' bottom and spreading through the whole width in function of the degree of stress
- after a 24 hour conditioning small cracks came out even for a 0.5 mm oversize.



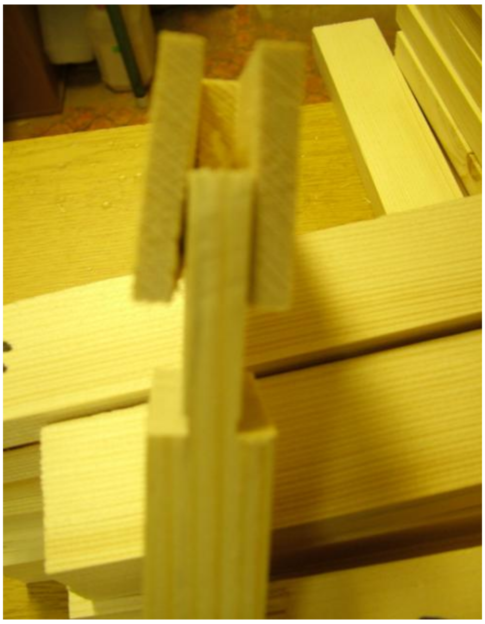


Figure 4-5. Experimenting with real specimens, model validation: assembly of joints

Table 2: Cracks observed in joints at assembly, and after 24 hour

Nr.	Width of tenon (mm)	Width of mortise (mm)	Interference (mm)	Final thickness of joint (mm)	Lenght of crack (mm)	Leght of crack after 24 hour (mm)
4	7,77	8,12	-0,35	24,16		
5	7,74	8,08	-0,34	24,08		
2	7,75	8,01	-0,26	24,03		
3	7,89	8,07	-0,18	24,16		
1	7,83	7,99	-0,16	24,11		
6	7,85	8	-0,15	24,17		
11	8,32	8,09	0,23	24,36		8,22
7	8,45	8,13	0,32	24,16		
12	8,35	8,02	0,33	24,16		16,21
10	8,36	8	0,36	24,72		14,09
9	8,41	7,98	0,43	24,96		17,48
8	8,63	8,05	0,58	25,19		26,01
14	8,97	8,05	0,92	25,87	32,28	32,29
15	8,93	8,01	0,92	25,76	21,79	21,85
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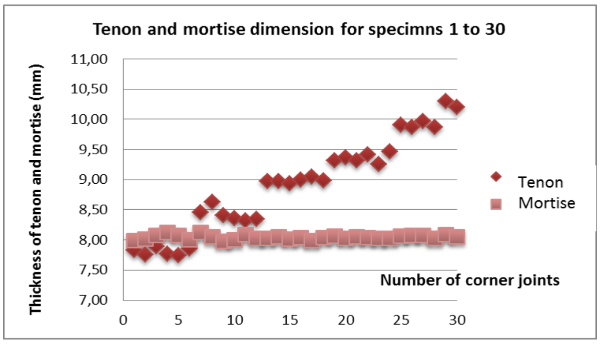

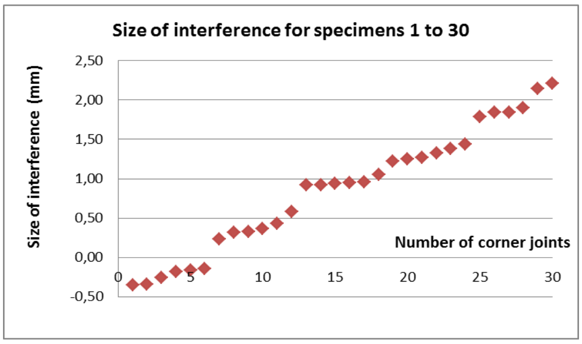

Figure 6: Changing interference size

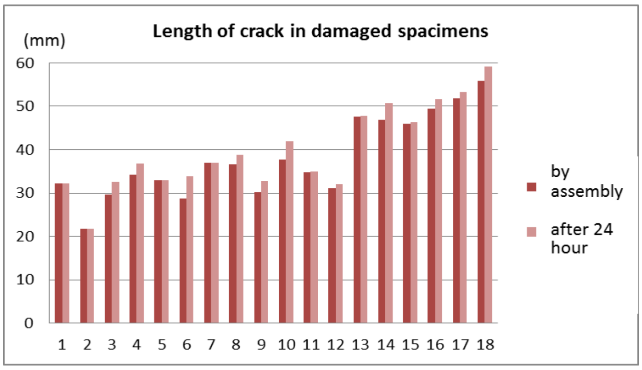

Figure 7: Change of crack length

It can be concluded that the size of cracks formed after assembling depends on the measure of overstressing. The results are affected additionally by factors such as moisture content, type of joint, total thickness, wood species and anatomical directions. The influence of these factors should be investigated individually however, based on the results the optimal pre-stressing level range between 0,0 and 0,5 mm.

Bonding properties

The joint assembly was prepared at a laboratory scale according to the glue utilization instructions. A PVA type glue, Emfibois 885 was used with a water resistance class of D2. After the joint gluing ad assembling the specimens were conditioned for 24 hours. The specimens were tested according to the MSZ EN 310 standard using an Instron 5566 type testing machine (Figure 8-9.). The measured strength values affirm the modelling results however, the reliability of the model is doubtful over the pre-stress range of 0,0 to 0,5 mm because of the presence of partial or full cross sectional cracks. These cracks first decrease the positive effect of pre-stressing on joint strength and completely

Figure 8-9. Experimenting with real specimens, model validation: strength test on bonded joints

Table 3: Load-bearing results of joints

Nr.	Sign of tenon	Sign of mortise	Interference (mm)	Deflection at failure (mm)	Max. bending strength (N)
3.	8/4	5	**-0,35**	-14.28	**910,5**
4.	8/5	1	**-0,34**	-11.23	**1078,4**
1.	8/2	3	**-0,26**	-12.46	**983,1**
2.	8/3	4	**-0,18**	-12.01	**947,6**
5.	8/6	6	**-0,15**	-34.43	**512,7**
9.	8,5/5	7	**0,23**	-10.33	**1002,7**
10.	8,5/6	8	**0,33**	-10.89	**950,8**
8.	8,5/4	10	**0,36**	-14.49	**1197,7**
7.	8,5/3	9	**0,43**	-15.54	**1492,1**
6.	8,5/2	12	**0,58**	-12.55	**937,4**

eliminate lately. In the case of tenons thicker with 1 mm than mortise just the gluing effect is measured. This underline the above mentioned optimal pre-stressing level range.

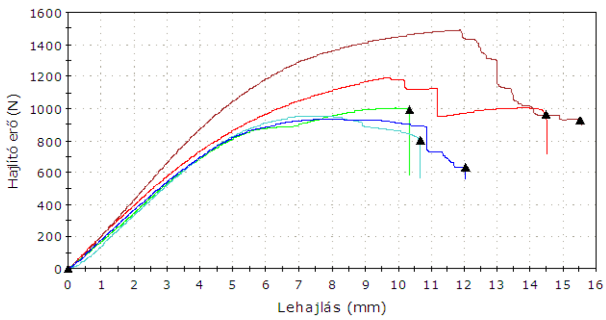

Figure 10. Typical load-deflection curves of specimens with interference (size of interference: 0.00-0.5 mm)

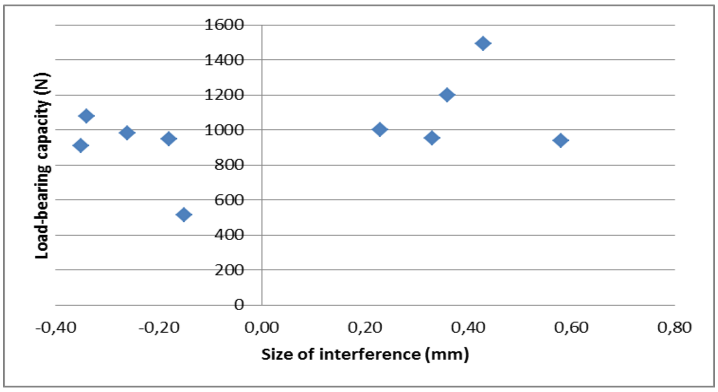

Figure 11. Load-bearing capacity

CONCLUSIONS

By virtual modelling of an open mortise and tenon joint with different amounts of interference it could be established that tensile stresses perpendicular to the grain developing around the mortise corner and concentrating in a very narrow area, though exhibiting themselves rather high as related to the corresponding strength values of wood give a good indication on how fit tightness becomes risky with increasing interference between tenon and mortise corner. A full evaluation of model results can be supported by physical experiments on joints.

The results of the experiments extended to testing the load-bearing capacity of the test-pieces prepared with different amounts of interference let us conclude that interferences between 0.0 mm and 0.25 mm result in higher strength of joints than when there is no interference (that is joints with clearance of 0.5 mm to 0 mm). However, it is known that the influence of interference on joint performance may be dependent on wood species and thickness of members that can be clarified with further investigations. As a consequence, the applicable fit tolerances and the corresponding tolerances on joint members can be derived depending on sizes and wood species.

REFERENCES

Koczor, Zoltán: *BEVEZETÉS A MINŐSÉGÜGYBE. A MINŐSÉGÜGY GYAKORLATI KÉRDÉSEI. (INTRODUCTION INTO QUALITY MANAGEMENT. PRACTICAL PROBLEMS, IN HUNGARIAN)* MŰSZAKI KÖNYVKIADÓ – MAGYAR MINŐSÉGI TÁRSASÁG BUDAPEST 2000

Kovács, Zsolt: *8.2. A MEGMUNKÁLÁS MINŐSÉGÉNEK BIZTOSÍTÁSA STATISZTIKAI FOLYAMATSZABÁLYOZÁSSAL IN: FAIPARI KÉZIKÖNYV II. (ASSURANCE OF MACHINING QUALITY THROUGH STATISTICAL PROCESS CONTROL PP. 415-435. IN WOOD HANDBOOK II.EDIT DR. MOLNÁRNÉ POSCH PAULA, IN HUNGARIAN)* FAIPARI TUDOMÁNYOS ALAPÍTVÁNY SOPRON 2002

Kemény, Sándor - Papp László – Deák András: *STATISZTIKAI MINŐSÉG- (MEGFELELŐSÉG-) SZABÁLYOZÁS. (STATISTICAL QUALITY (CONFORMITY) CONTROL, IN HUNGARIAN)* MŰSZAKI KÖNYVKIADÓ – MAGYAR MINŐSÉGI TÁRSASÁG BUDAPEST

Sitkei, György szerk.*: A FAIPARI MŰVELETEK ELMÉLETE. (THEORY OF WOOD PROCESSING OPERATIONS, IN HUNGARIAN)* MEZŐGAZDASÁGI SZAKTUDÁS KIADÓ KFT. BUDAPEST. 1994

Bódig, József – B. A. Jayne: *MECHANICS OF WOOD AND WOOD COMPOSITES, VAN NOSTRAND REINHOLD PUBLISHING COMPANY INC., NEW YORK 1982*

Bariska, Mihály – Molnár, Sándor: WOOD SPECIES OF HUNGARY, SZAKTUDÁS KIADÓ HÁZ RT., BUDAPEST, 2002

CORRESPONDING AUTHOR:

Tibor PAPP
Institute of Product Development and Manufacturing, Faculty of Wood Sciences, University of West Hungary
4. Bajcsy-Zsilinszky str., Building D.
9400 Sopron, Hungary
phone: +36/99 518-178 fax: +36/99 518-300 e-mail: papptyby@fmk.nyme.hu