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OPTIMIZATION MODEL OF MAINTENANCE OF LIGHT INDUSTRY ENGINEERING STRUCTURES

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Abstract:

In my PhD [1]dissertation optimization models were developed for the pavement management system, which could help planning the optimal maintenance work. The deterioration model is based on the Markov transition probability matrix.

The maintenance and rehabilitation planning process which is used in this PMS model can be used in case of several engineering structures. This model can be generalized for those engineering structures, where there are several types of structures and for each structure there are many components with many different types of deterioration options and duty uses. This paper summarizes the conditions of usage and the element of the algorithm.

In this paper we would like to present the development of Engineering Structures Management System, the different models, and furthermore a practical application – the deterioration model of parts of light industry engineering structures and the planning of their maintenance.

Keywords:

deterioration model, pavement management, optimization, light industry

1 INTRODUCTION

The engineering structures maintenance and repair needs usually far exceed the resources available to address these needs, so many firms have to turn to the development of ESMS (Engineering Structures Management System). The ESMS is a method to improve the allocation of these limited resources and the condition of their engineering structures.

The ESMS refers to the careful allocation of funds available for these purposes: maintenance, repair and rehabilitation to ensure that the funds are used in their most effective way. Specifically an ESMS is a rational and systematic approach to organizing and carrying out the activity related to planning, designing, constructing and replacing structures.

The one part of the total ESMS is a computer program. The software associated with an ESMS should provide the following functions:

- A database which contains the necessary information needed for this purposes including inventory and inspection data, and information related to maintenance, repair, and rehabilitation actions and effectiveness. This base contains historical data (deterioration past and future maintenance and rehabilitation actions, cost, etc.).
- A mayor maintenance, rehabilitation and replacement component, which contains the actions and its prices, etc.
- Heuristic or optimization procedure which gives the cheapest maintenance and rehabilitation actions.
- Deterioration model which determine the future condition state of an engineering structure depending on the actual condition state and the time period.

A deterioration-based model was presented for Road Management in [2].

2 DETERIORATION MODELS

The quality of an engineering structure (or part of it) usually does not remain constant over time. It deteriorates throughout the service of life. The service of life can be estimated from [3]:

- empirical experience,
- database which contains time series,
- using performance models,
- laboratory testing.

We can group the used methods into two basic types: deterministic and probabilistic. In the deterministic models the condition is predicted as a precise value on the basis of mathematical functions of observed or measured deterioration. This class includes several approach: mechanistic, regression, mechanistic-empirical, etc. In the probabilistic models the condition is predicted as a probability function of a range of possible conditions.

Two type of probability functions can be used [4], the probability distribution and the Markov transition distribution matrix.

The probability distribution is a continuous function.. This shows the probability of condition index being greater than a given value in relation to the age of the structure. This type of function is sometimes known as a survivor curve.With the help of that probability distribution function we can determine the deterioration function, too.

The initial condition and the quality are near to the top of the scale. That is near 100%. A minimum acceptance level must also be set. It will vary with many factors, such as class of facility, agency policy, safety and economics. A maintenance or rehabilitation action at any time cans extent the service of life at a facility.

For the determination of the deterioration curve serial time series is needed. If it is not available, we use the Markov matrix. In our model we used this matrix.

To create the matrix first we divide the possible conditions into discrete condition states. For example the condition state can be divided into 10 stages: 10%, 20%, 30%, …100%. At any point in time probabilities are given for the likelihood of the structure being in each condition states and there are defined in a “transition matrix”. This matrix is used to predict the condition state after a time period (year, 2 years or more years). A matrix is shown in Table 1.

Let us denote this matrix by $\mathbf{Q}$. The q_{ij} element of matrix $\mathbf{Q}$ is the transition probability from state i to state j. This structure assumes that the conditions after a time period state or deteriorate to some lower state. Several methods are known to determine the matrix $\mathbf{Q}$. The most commonly used approach for estimation of transition probabilities is the linear regression method, but it can be determined by a Poisson regression model or by negative binominal regression.

Table 1: Markov transition probability matrix

Initial condition state	Probability of archiving future condition state									
	1	2	3	4	5	6	7	8	9	10
1:100%-91%	0.95	0.03	0.02							
2:90%-81%		0.93	0.07							
3:80%-71%			0.94	0.04	0.02					
4:70%-61%				0.95	0.05					
5:60%-51%					0.92	0.05	0.03			
6:50%-41%						0.96	0.04			
7:40%-31%							0.95	0.04	0.01	
8:30%-21%								0.93	0.07	
9:20%-11%									0.95	0.05
10:10%-0%										1

3 OPTIMIZATION MODEL

We can apply the deterioration model of ESMS for the pavement management system, for the deterioration and maintenance of car parts [1] and we can use it in the light industry, too. The condition of application is: the deterioration of certain components not only depends on time, but on the extent of use.

In our model the following denotation will be used

- The type of the engineering system: **tip** ($tip_1, \ldots, tip_m, \ldots, tip_M$) – m is the type index, M is the number of types.
- The number of each machine types contents the **db** vector $(db_1, db_2, \ldots, db_M)$.
- The number of different parts of the certain machines signs N. We can suppose that the number of parts at the machine types is equal. Here we see the parts not as elementary parts, but bigger, common repairable unit. Sign r_{mn} the number of n-th parts of m-th types machine. These means, we have $db_m \cdot r_{mn}$ pieces of such part altogether.
- The parts can be in different states. The condition of a part is described by different types of deterioration parameters (for example visual characterization, result of instrumental monitoring). Sign the number of states parameters with S. Note 1 denotes the best, and note 4 denotes the worst condition: perfect – note 1, fault, does not disturb the normal use – note 2 , fault, disturbs the normal use – note 3, useless – note 4.
- The altering of the condition, the deterioration depends on the other factors – for example efficiency– too. Denote the number of different values of theses F, the adequate index let be f.
- The possible maintenance types are denoted with vector **p** $(p_1, \ldots, p_k, \ldots, p_K)$ – k is the index, K is the number of possible maintenance operations. The intervention can be simple repairing, the change of a smaller unit, the change of a bigger unit or the change the machine part.
- In case of the multiperiod algorithm the index of year is t, the number of years is T.

The size of the vectors and matrices depends on the number of parameters and the different values of parameters. If the number of parameters is *S* and the number of different notes is 4, then the number of possible stage is S^4, for example if $S = 4$ than 4^4=256. It means, the size of vectors is 256, the number of elements of matrix is $256 \cdot 256$. Denote the size of matrix *L*.

This number determines the size of the unknown variable vector **X**, the elements of this vector denote proportion. The number of **X** vectors is $M \cdot N \cdot F \cdot K \cdot T$.

The *l*-th element of vector $\mathbf{X_{mnfkt}}$ shows the machine parts belonging to *m, n, f, k, t* indexes and is in the *l*-th condition, that how many percent of this part the p_k maintenance must be realized. This element is x^l_{mnfkt}, or $(\mathbf{X_{mnfkt}})^l$. We use an upper index, if we refer an element of vectors or matrices.

Let us denote the Markov transition probability matrix by $\mathbf{Q_{mnfk}}$ which belongs to *n*-th part of the *m*-th type, the *f*-th efficiency and the *k*-th maintenance action. The number of different matrices is $M \cdot N \cdot F \cdot K$, the number of rows and columns of matrices is the number of quality condition stages, *L*. The *l*-th element of *i*-th row of $\mathbf{Q_{mnfk}}$ matrix is q^{il}_{mnfk}, or $(\mathbf{Q_{mnfk}})^{il}$ gives the probability that if the machine parts which at the starting time is in *i*-th stage, at the end of the planning period get into the *l*-th condition stage.

Let us denote the unknown vector $\mathbf{V_{mnft}}$, that gives the fraction of those engineering systems which belongs to the *n*-th parts of the *m*-th type, to the *f*-th efficiency at the end the *t*-th time period.

Let us denote the vector $\mathbf{b_{mnf}}$, which gives the fraction of the different condition stages of the *n*-th parts of the *m*-th type, belong to the *f*-th efficiency at the starting time of planning.

There are several conditions to fulfill. The first condition (1) is related to the fraction of the engineering systems at the initial year:

$$\sum_{k=1}^{K} \mathbf{U}\mathbf{X_{mnfk1}} = \mathbf{b_{mnf}}, \quad m = 1,2,\ldots,M \quad n = 1,2,\ldots,N \quad f = 1,\ldots,F \tag{1}$$

where **U** is $L \cdot L$ size unit-matrix. We must choose such an **X** vector in first year, which gives the starting $\mathbf{b_{mnf}}$ vector in case of all machine types, all parts and all efficiency.

The second condition defines the vector $\mathbf{V_{mnf1}}$ (2), the fraction of structures, of the machine parts at the end of the first planning:

$$\sum_{k=1}^{K} \mathbf{Q_{mnfk}}\mathbf{X_{mnfk1}} = \mathbf{V_{mnf1}}, \quad m = 1,\ldots,M \quad n = 1,\ldots,N \quad f = 1,\ldots,F \tag{2}$$

The next condition (3) applies to the mediate years. This means, that the $\mathbf{V_{mnft}}$, the fractions at the end of *t*-th time period gives the starting distribution for the (*t*+1)-th period. For each year the following conditions must be fulfilled:

$$\sum_{m=1}^{M}\sum_{n=1}^{N} \mathbf{U}\mathbf{X_{mnfk(t+1)}} - \mathbf{V_{mnft}} = \mathbf{0}, \quad f = 1,\cdots,F \quad k = 1,\ldots,K \quad t = 1,\ldots,T-1 \tag{3}$$

This condition defines the unknown $\mathbf{X_{mnft}}$ vector.

One of the maintenance policies has to be applied (4) for every structure in each year:

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}\mathbf{X}_{\mathbf{mnfkt}}=\mathbf{1},\quad t=1,\ldots,T\,. \tag{4}$$

The machine parts are divided into 3 groups: acceptable (good), unacceptable (bad) and the rest. Let us denote the three set by G the good, R the bad and E the set of other structures and by H the whole set of structures. The following relations are realizing to the sets:

$$\begin{aligned}&G\cap R=\varnothing \qquad G\cap E=\varnothing\\&R\cap E=\varnothing\\&G\cup R\cup E=H\end{aligned} \tag{5}$$

The following conditions (6) are related to these sets in the initial year

$$\begin{aligned}&\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l}\geq\alpha_1\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}(\mathbf{b}_{\mathbf{mnf}})^{l},\ \ l\in G\\&\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l}\leq\alpha_2\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}(\mathbf{b}_{\mathbf{mnf}})^{l},\ \ l\in R\text{'}\\&(\underline{\mathbf{b}}_E)^{l}\leq\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l}\leq(\overline{\mathbf{b}}_E)^{l},\quad l\in E\end{aligned} \tag{6}$$

where

- G, R, E are given above,
- $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}(\mathbf{b}_{\mathbf{mnf}})^{l},\ \ l\in G$ the fraction of structures in the good set before the planning period,
- $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l},\quad l\in G$ the fraction of structures in the good set after the first year,
- $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}(\mathbf{b}_{\mathbf{mnf}})^{l},\ \ l\in R$ the fraction of structures in the bad set before the planning period,,
- $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l},\quad l\in R$ the fraction of structures in the bad set after the first year
- $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfk1}})^{l},\quad l\in E$ the fraction of structures in the other set after the first year,
- $\underline{\mathbf{b}}_E$ the lower bound vector for the fraction of structures in the other set,
- $\overline{\mathbf{b}}_E$ the upper bound vector for the fraction of structures in the other set,
- α_1 and α_2 given constants.

The first condition means that the amount of the machine parts in the “Good” set must be more or equal than a given value, in this case this is proportional with the starting quantity. The second condition does not allow that after the first year the amount of the parts in the “Bad” set can be more than a certain percent of the starting amount. The third condition gives lower and upper bound for the other parts after first year.

For the further years similar inequalities (7) could be used

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\mathbf{Y}_{\mathbf{ijt}} \ \mathbf{R} \ \sum_{i=1}^{I}\sum_{j=1}^{J}\mathbf{Y}_{\mathbf{ij(t+1)}}, \quad t=1,2,\ldots,T-1 \tag{7}$$

where **R** could be one of the relations <, >, =, <=, >= and these relations could be given in connection with each condition states (e.g. each rows could have different relations).

Instead of (6) and (7) condition states could be applied for the end (8) of the planning period (e.g. for $t=T$) :

$$\begin{aligned}
&\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfkT}})^{l} \geq \alpha_1\sum_{i=1}^{I}\sum_{j=1}^{J}(\mathbf{b}_{\mathbf{mnf}})^{l}, \quad l \in G\\
&\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfkT}})^{l} \geq \alpha_2\sum_{i=1}^{I}\sum_{j=1}^{J}(\mathbf{b}_{\mathbf{mnf}})^{l}, \quad l \in R\\
&(\underline{\mathbf{b}}_{E})^{l} \leq \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}(\mathbf{Q}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfkT}})^{l} \leq (\overline{\mathbf{b}}_{E})^{l}, \quad l \in E
\end{aligned} \tag{8}$$

Beside the condition for the states at all maintenance exercise the cost factor is very important. As far as possible we must work such a maintenance strategy, which fulfills the conditions for the states and it has the lowest cost.

Let us denote by vector $\mathbf{C}_{\mathbf{mnfk}}$ the unit cost vector of the maintenance policy k, belongs to the n-th part of the m-th type in case of f-th efficiency. The elements of vectors show that in case of certain qualification state how much would it cost to make a unit of the maintenance action.

We can formulate more different conditions in connection with costs. One of them is the yearly budget bound of each maintenance action:

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}r^{(t-1)}\mathbf{C}_{\mathbf{mnfk}}\mathbf{X}_{\mathbf{mnfkt}} = r^{(t-1)}M_k, \quad t=1,\ldots,T \quad k=1,\ldots,K \tag{9}$$

where r is the interest rate, $\mathbf{C}_{\mathbf{mnfk}}$ is the unit cost vector of the maintenance policy k, of the n-th part of m-th type, belong to f-th efficiency and M_k is the budget bound available for maintenance policy k in the initial year.

Now the objective of the problem is formalized. The objective (10) is to minimize the total cost of maintenance:

$$C = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{t=1}^{T}\mathbf{X}_{\mathbf{mnfkt}}\mathbf{C}_{\mathbf{mnfk}} \rightarrow MIN\ ! \tag{10}$$

If the yearly available B sum is given, then we can formulate two further conditions in connection with the budget bound.

The budget bound condition for the initial year:

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}\mathbf{X}_{\mathbf{mnfk1}}\mathbf{C}_{\mathbf{mnfk}} \le B \ . \tag{11}$$

The conditions for further $t = 2,3,\ldots,T$ years are the following (12):

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K} r^{(t-1)}\mathbf{X}_{\mathbf{mnfkt}}\mathbf{C}_{\mathbf{mnfk}} \le r^{(t-1)}B \ . \tag{12}$$

Besides the minimization of the maintenance costs we could aim to minimize the user's costs. The user's costs depend on the type of machine, on the parts and on the efficiency. Let us denote this user's cost-vector by $\mathbf{K}_{\mathbf{mnf}}$. The *l*-th coordinate of the vector belongs to the *l*-th qualification state. ($1 \le l \le L$, in our example it is 256).

The objective function of the minimization of users costs is the following (13):

$$C = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{t=1}^{T}\mathbf{V}_{\mathbf{mnft}}\mathbf{K}_{\mathbf{mnf}} \to MIN \ ! \tag{13}$$

Often it is practical to handle the two type of costs together and to write a combined objective function. We can do this the following way (14):

$$C = \alpha\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{t=1}^{T}\mathbf{X}_{\mathbf{mnfkt}}\mathbf{C}_{\mathbf{mnfk}} + \beta\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{f=1}^{F}\sum_{t=1}^{T}\mathbf{V}_{\mathbf{mnft}}\mathbf{K}_{\mathbf{mnf}} \to MIN \tag{14}$$

Here we optimize in case $\alpha = 0$ only the user's costs, in case $\beta = 0$ only the maintenance costs. So we can decide arbitrary objective functions with the different value of two parameters.

4 CONCLUSIONS

I think, this mathematical model is adaptable to the light industry, too. For example in the printing industry and in the paper and packing industry we can investigate the adaptability of this model. In [5] is shown the models of maintenance in printing industry.

Nowadays, key printing machines making up the technological lines are planned for lifetimes of approx. 20 years with respect to the two-shift work order that is customary in Western Europe.

In actual practices, reaching half of this lifecycle (first cycle) machines are subjected to full-scaling reconditioning and modernization. The underlying reason is that in comparison to other industries this sector sees a large turnover of used machines. Printing houses tend to plan the financial return of any printing machine for a single cycle, and instead of general reconditioning they rather sell the equipment, and buy a new or reconditioned one for the following reasons:

- It is difficult to spare these large-capacity machines for the duration of reconditioning.
- In general, printing houses find it difficult to create proper circumstances for expert reconditioning.
- Machines are replaced to follow technical, technological development.

The associated examples show that the above statements are true for all the printing machines, and not only technological lines. In recent years, Hungarian practices have followed these tendencies. It is to be also noted that Hungarian printing industry seems to be an enthusiastic buyer of equipment that has been over the first phase of their lifecycles, and thus dismounted in Western European printing houses.

In the light of the foregoing, the cycle structure shown in *Figure 1* can be planned for the equipment of printing and finishing (binding) operations:

P – R – M – R – S – R – M – R – G – R – M – R – S – R – M – R – D(G)

2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18.

> > > > > > > > > > > > **sequence of years** > > > > > > > > > > > >

P – procurement, guarantee period	R – review, quick servicing
G – general servicing	S – small repairs
D – scrapping, de-commissioning	M – medium repairs

Figure 1: Servicing cycle structure of printing machines

If we analyze the repair actions (in case of the small, medium repairs, at the quick and the general servicing), its costs, on the basis of observations we can decide the Markov transition probability matrix.

The adaptation of this mathematical model can help us in the economic planning of maintenance and rehabilitation actions in the printing industry.

REFERENCES

[1] Ambrusné Somogyi, K.: ÚTBURKOLAT-GAZDÁLKODÁS VÁLTOZÓ FELTÉTELEK MELLETTI OPTIMALIZÁCIÓS MODELLJEI, *Doktori (Ph.D.) értekezés, Széchenyi Egyetem, 2009.*

[2] Ambrus-Somogyi, K.: DETERIORATION-BASED MAINTENANCE MANAGEMENT ALGORITHM, *Acta Polytechnica Hungarica* – Vol. 4 No.1. 2007. pp. 119-126. *ISSN 1785-8860*

[3] Hudson, W. R., Haas, R., Uddin, W.: INFRASTRUCTURE MANAGEMENT, *Mc Giraw-Hill P.C.*, 1997, pp. 393.

[4] Ambrus-Somogyi. K., Bakó, A.: PERFORMANCE MODELING OF STRUCTURES – *Proceedings of IN-TECH-ED'05 Conference BMF*, Budapest 2005. szeptember 7-9. pp. 389-398.

[5] Horváth, Cs.: MODELS AND KEY FACTORS OF MAINTENANCE IN PRINTING INDUSTRY, *Proceedings of 13th Blaz Baromic International Conference on Printing, Design and Graphic Communications*, Senj, September 30th – October 2nd, 2009 Invited Lecture, pp. 10-16

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